Heavy Meson Spectroscopy at $\beta = 6.0$

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We present results of a quenched calculation of the heavy-light and quarkonium spectrum using the tadpole improved clover action. We resolve completely the triplet χ P-states in quarkonium systems, and obtain evidence for fine structure of the heavy-light P-states. Approximate scaling of the hyperfine splittings is observed, producing results that are significantly below experiment.

1. Introduction

Charm physics continues to pose difficulties for lattice QCD simulations. The systems are significantly relativistic [1,2] causing problems for the NRQCD approach, and have $am_Q \simeq O(1)$ causing significant discretisation effects in the heavy Wilson quark approach. We present the results of a simulation using the tadpole improved clover action and perform the analysis using the Fermilab [3] interpretation of the heavy Wilson quark approach.

2. Simulation Details

The simulation was performed using 499 quenched gauge configurations on a $16^3 \times 48$ lattice. Five heavy quark masses and three light quark masses, detailed in Table 1, were simulated using the tadpole improved clover action, with $u_0 = 0.8778$ from the average plaquette, and $C_{SW} = 1.47852$. Both local and fuzzed [4] operators were generated at source and at sink. (Local) covariant derivative sources in each of the spatial directions were used for the $\kappa = 0.12600$ quark corresponding to $\kappa_{\rm charm}$, allowing the operator for a $^3P_2$ state to be created for combinations involving $\kappa = 0.12600$ with each of the other masses. The operators used are given in Table 2.

Table 1
$\beta = 6.0$ simulated kappas

κ	aM_{PS}	Fuzzing Radius
0.13856	0.228(2)	6
0.13810	0.293(1)	6
0.13700	0.4135(10)	6
0.13000	0.9283(7)	3
0.12600	1.1618(6)	3
0.12200	1.3755(6)	3
0.11800	1.5751(6)	3
0.11400	1.7644(6)	3

Table 2
Meson operators

State	J^{PC}	Operators	
$^1S_0$	0^{-+}	$\bar\psi\gamma_5\psi$	
$^3S_1$	1^{--}	$\bar\psi\gamma_i\psi$	
$^1P_1$	1^{+-}	$\bar\psi\sigma_{ij}\psi$	
$^3P_0$	0^{++}	$\bar\psi\psi$	
$^3P_1$	1^{++}	$\bar\psi\gamma_i\gamma_5\psi$	
$^3P_2$	2^{++}	$\bar\psi\{\gamma_i\Delta_i - \gamma_j\Delta_j\}\psi$	E rep
		$\bar\psi\{\gamma_i\Delta_j + \gamma_j\Delta_i\}\psi$	T rep

An extensive analysis of correlated double and single exponential fits to various smearing combinations was carried out, and the optimal fitting approach selected for each channel in the light-light, heavy-light and heavy-heavy sectors independently.

3. Fine Structure

We obtain a signal for the fine structure of the χ triplet of P-states in quarkonium, as illustrated in the effective mass plots in Figure 1. In the heavy-light case we obtain a signal for the splitting between the 1^+ and 0^+ correlators, Figure 2, fitted using a single exponential model for the fuzzed-fuzzed combination. The consistency of both double and triple exponential multi-correlator fits was checked. Cross correlations between the 1^{++} and 1^{+-} operators showed that there was significant mixing, as expected since the $j_{\rm light}$ basis is

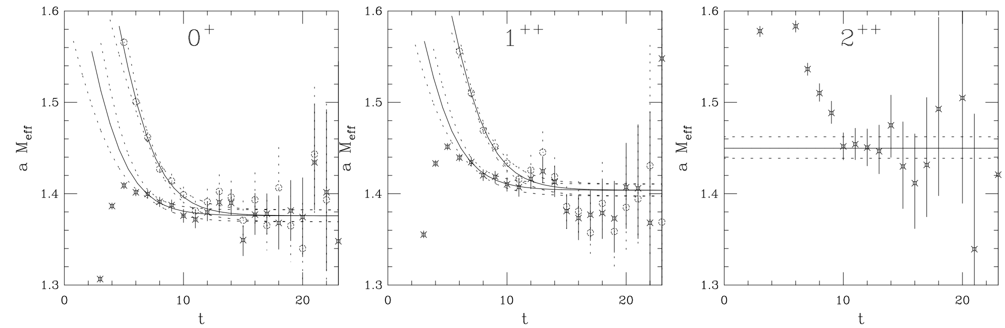
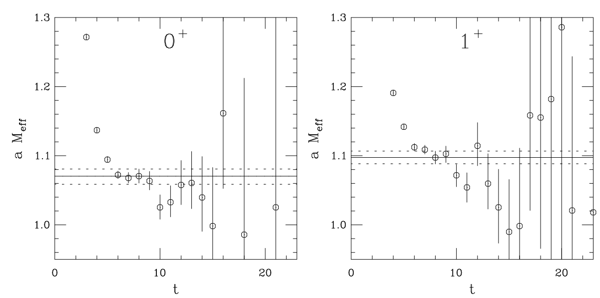


Figure 1. Quarkonium χ triplet for the ($\kappa = 0.12600, \kappa = 0.12600$) combination. Double exponential fits were performed to the fuzzed-local and local-local correlators simultaneously for the 0^{++} and 1^{++} states, while a single exponential fit was performed to the derivative based operator for the 2^{++} state. The three states are clearly resolved.

Figure 2. Heavy-light fine structure for the ($\kappa_Q = 0.12600$,$\kappa_q = 0.13810$) combination; fit is to timeslices 7-13 of the fuzzed-fuzzed correlator.



the physical basis in the $m_Q \to \infty$ limit. However we could not resolve the two physical 1^+ states, and treat the ground state for each of the 1^+ correlators as the lower lying 1^+ state, assumed to have $j_{\rm light} = \frac{1}{2}$.

4. Spectrum at $\beta = 6.0$

We use the dispersive mass m_2 of pseudoscalar mesons defined by $E(p^2) = m_1 + \frac{p^2}{2m_2} + Cp^4$, as the definition of the heavy quark mass in extrapolations of the spectrum. We found that linear extrapolations in the inverse heavy quark mass gave a very acceptable χ^2/dof. For the heavy-light systems, linear chiral extrapolations were performed to $\kappa_{\rm crit}$ and to $\kappa_{\rm strange}$, followed by linear extrapolations in the inverse heavy-strange pseudoscalar mass to the D_s and B_s masses.

For the $^3P_2$ state, where only combinations of propagators involving the $\kappa = 0.12600$ quark (with another) could be formed from the available data, the non-degenerate combinations were used to extrapolate to the physical meson masses, introducing a small correction in extrapolations to the J/ψ system. However, the extrapolations of the $^3P_2$ splittings to Υ are not well under control. The results obtained using the string tension, m_ρ and the quarkonium $S-P$ splitting to set the scale, and the kaon mass to fix $\kappa_{\rm strange}$, are tabulated in Table 3 [1] and Table 4.

5. Scaling Behaviour

Comparison to a calculation using 220 configurations at $\beta = 6.2$ on a $24^3 \times 48$ lattice [6] with the same action, allows some estimate of the scaling behaviour to be made. We plot the lattice spacing dependence of the charmonium and D_s hyperfine splittings in Figure 3.

Near-scaling behaviour is seen with both the string tension and with m_ρ used to set the scale. Scaling is not seen with the quarkonium S-P splitting. However, the lack of scaling is only a 1σ effect. NRQCD calculations [1] have found the Υ S-P splitting scaling well with m_ρ, at about

[1] There is a systematic uncertainty of order 50 MeV [5] in the values for the heavy-light S-P splitting since the experimental values are for the $j_{\rm light} = \frac{3}{2}$ doublet, while we calculate $j_{\rm light} = \frac{1}{2}$ states. For the heavy-light P-states we have adopted the nomenclature used for the $j_{\rm light} = \frac{1}{2}$ doublet in the Kaon system by the PDG.

Table 3
$\beta = 6.0$ heavy-light mass splittings (MeV)

Scale	$\sqrt{K}$	M_ρ	S-P	Expt
$D^* - D$	110(7)	106(8)	129(10)	142
$D_s^* - D_s$	99(5)	95(6)	115(9)	144
$\bar{D}_s - \bar{D}$	98(5)	96(5)	107(6)	105
$D_1 - \bar{D}$	540(30)	530(30)	600(30)	459
$D_{s1} - \bar{D}_s$	494(18)	480(20)	545(20)	460
$D_1 - D_0^*$	45(20)	44(20)	47(25)	-
$D_{s1} - D_{s0}^*$	57(12)	56(11)	64(13)	-
$B^* - B$	41(9)	39(10)	59(11)	46
$B_s^* - B_s$	40(6)	38(7)	57(8)	47
$\bar{B}_s - \bar{B}$	90(6)	88(6)	110(7)	91
$B_1 - \bar{B}$	490(30)	480(30)	590(40)	419
$B_{s1} - \bar{B}_s$	440(20)	430(20)	540(30)	446
$B_1 - B_0^*$	50(20)	50(20)	60(20)	-
$B_{s1} - B_{s0}^*$	43(11)	42(10)	54(12)	-

Table 4
$\beta = 6.0$ heavy-heavy mass splittings (MeV)

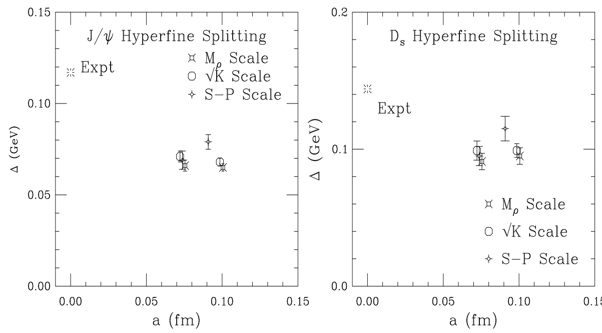
Scale	$\sqrt{K}$	M_ρ	S-P	Expt
$J/\psi - \eta_c$	68(2)	65(2)	79(4)	117
${}^1P_1 - \bar{S}$	418(13)	408(16)	-	458
$\chi_{c2} - \chi_{c1}$	81(28)	78(27)	93(33)	46
$\chi_{c1} - \chi_{c0}$	51(7)	49(7)	59(11)	95
$\chi_{c2} - \chi_{c0}$	133(28)	128(28)	153(35)	141
$\Upsilon - \eta_b$	22(1)	21(1)	31(3)	40
${}^1P_1 - \bar{S}$	366(17)	358(20)	-	460
$\chi_{b2} - \chi_{b1}$	43(30)	42(30)	56(32)	21
$\chi_{b1} - \chi_{b0}$	26(9)	25(9)	33(10)	32
$\chi_{b2} - \chi_{b0}$	65(30)	63(27)	85(32)	53

30% below its experimental value. This suggests that ultimately scaling with respect to the S-P splitting will be seen at values of the hyperfine splittings above those with m_ρ and the string tension. Likewise we find the D and Υ hyperfine splittings to show approximate scaling below experiment with m_ρ and the string tension. After extrapolating the heavy-light results to B and B_s, the errors are such that our values are consistent with experiment.

6. Conclusions

We resolve completely the triplet of χ states in the J/ψ system with a relativistic action, and obtain evidence for fine structure in heavy-light systems. We find that both the quarkonium and D hyperfine splittings scale well with both the string tension and with m_ρ, lying significantly below their experimental values. We find that our results using the quarkonium S-P splitting do not scale well. This may be due to the use of only local propagators in the quarkonium calculation at $\beta = 6.2$, (which plateau at large t, making the plateau identification and statistical noise touble-some), or to discretisation effects. Further calculations (better smearing at $\beta = 6.2$ and possibly at higher β) are required to resolve which is the case.

Figure 3. Scaling Behaviour of hyperfine splittings.



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